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Weinberg sum rules and the parity doubling of radial Regge trajectories

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ABSTRACT

Assuming that the radial excitations of $\bar{q}q$ meson states exactly follow linear Regge trajectories with constant residues, as prescribed by dual models, and using large N_c arguments and the matching to perturbative QCD in the deep-Minkowski region we obtain that: (a) the region dominated by resonances and the one saturated by perturbation theory approach each other as N_c increases; (b) the scales $\Lambda^{(V,A)}$ separating the resonance-dominated and the perturbative-saturated region in the V, A channels, respectively, grow as $\sqrt{N_c}$, whereas the difference between $(\Lambda^V)^2$ and $(\Lambda^A)^2$ stays constant or even decreases as N_c increases; (c) the number of visible resonances increases as N_c; (d) in order to satisfy the Weinberg sum rules the slopes of Regge trajectories for mesons of opposite parities must coincide, but the intercepts may differ by a quantity of $\mathcal{O}(1)$ at most in the large N_c limit. This suggests that modelizations of QCD where these characteristics are not present and yet the resonances follow linear Regge trajectories are not compatible with the symmetries or short-distance properties of QCD.

1. Introduction

Recently the issue as to whether radial excitations of mesons with a given spin but of opposite parities become eventually degenerate in mass in the large N_c limit has been hotly debated [1–17]. The conclusions of different authors diverge as to whether chiral symmetry restoration at high energies implies that meson masses asymptotically approach each other [1–10], or, on the contrary, the footprint of chiral symmetry breaking persists for arbitrarily high mass mesons (in the large N_c limit) [11–14]. Some results based on AdS/QCD correspondence [15,16] have also questioned whether the slopes of Regge trajectories for mesons of opposite parities should be equal. While this issue has a relatively long history (reviewed recently in [7,17,18]), the recent interest has been fuelled by improvements in meson phenomenology [18–21].

It should be declared at the outset that all the previous works and ours too are framed in the context of exact linear Regge trajectories with constant residues. Slopes, residues and intercepts may depend on the J^{PC} quantum numbers of the resonances, but once these are fixed the whole asymptotic spectrum can be predicted. This is what dual models teach us. We are not going to prove or disprove dual models here. We shall *assume* that they describe sufficiently well the asymptotics of QCD (and there is ample evidence for that). This is the arena of the discussion that is followed by all previous work on this subject without exception.

To be precise the problem under discussion is the following. If we *assume* that in QCD the trajectories of hadrons follow linear Regge trajectories, what has chiral symmetry to say about the slopes and intercepts of channels that have identical quantum numbers but parity? Obviously if chiral symmetry was not broken, the $SU(N_f) \times SU(N_f)$ chiral symmetry would guarantee complete degeneracy. The issue is then how the spontaneous breakdown of chiral symmetry gets reflected in the spectrum. In fact, we are interested only in the asymptotic spectrum for large values of the radial number n in specific channels that differ in parity.

Understanding this issue is interesting for several reasons. One of them is that many physical observables (such as e.g. the mass difference between charged and neutral pion masses, see e.g. [5]) can be expressed as an infinite sum over resonances. It would be nice to be able to cut-off this infinite sum due to exact cancellation between the channels of opposite parity. In addition, understanding the consequences of chiral symmetry breaking on the asymptotics of Regge trajectories would be very useful to disentangle the large n behavior from the manifestation of chiral symmetry breaking in the lowest lying resonances.

This problem has been studied traditionally by considering the $N_c \to \infty$ limit (where QCD is described by an infinite set of narrow resonances) at the outset and using the Operator Product Expansion (OPE) in the Euclidean region, but extracting conclusions is hampered by the fact that the sums over the infinite number of resonances are ill defined. Thus a pessimistic point of view seems to have been taken by several authors according to which one cannot actually prove anything.

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Actually, this is not true as the following analysis reveals. The new ingredients of the present work are:

(a) keeping N_c large but finite, as this is very useful to keep under theoretical control the "crossover" between the region of resonance saturation and the high energy region where perturbative QCD is valid. The physical approach to the large N_c limit teaches us how to handle the infinite sums that would otherwise appear;
(b) in order to deal with resonance physics we shall work consistently in Minkowski momentum space, as opposed to previous analysis published in the literature;
(c) using the Weinberg sum rules [22,23] in the form which follows from the Wilson OPE in the coordinate Minkowski space and asymptotic freedom.

The framework of the present discussion is phenomenological Regge theory. As stated we shall assume without further question that all hadrons fall into Regge trajectories. These trajectories will be assumed to be exactly linear. Obviously we know that there are large deviations from linearity for the lightest resonances. In fact in order to eventually understand these deviations from QCD and chiral symmetry breaking considerations we need to have theoretical control on the large n behavior. In this Letter we are only concerned about the latter. We stress the difference in our analysis from what has been recently done. Namely, in the thorough analysis undertaken in [11–14] the equality of intercepts of Regge trajectories and of residues of resonances has been assumed for granted whereas just the opposite assumption was taken also for granted in [15,16]. Our goal is to exploit the model independent spectral sum rules of Weinberg time at large but finite number of colors N_c and to use few facts justified by phenomenology of chiral symmetry breaking in order to show that spontaneous chiral symmetry breaking indeed entails the equality of intercepts of Regge trajectories and of residues of resonances in two point correlators for parity-partner sets of mesons.

2. Large (but finite) number of colors

Let us consider the correlators

$$\Pi^j_{\mu\nu}(x,y) = \big\langle T\big(J_\mu(x) J_\nu(y)\big)\big\rangle \equiv (-i)\int \frac{d^4p}{(2\pi)^4} \exp\big[ip(y-x)\big]\Pi^j_{\mu\nu}\big(p^2\big), \quad j = V, A,$$
$$J^j_\mu = \big(\bar q(x)\gamma_\mu q(x), \bar q(x)\gamma_5\gamma_\mu q(x)\big). \tag{1}$$

As we keep the chiral limit in the major part of our analysis we do not need to specify the internal symmetry group and we omit the flavor indices. The color degrees of freedom of the quark fields $\bar q, q$ are also omitted in the notation.

For finite N_c two different, non-overlapping, regions of physics can be clearly identified in the physical (Minkowski) momentum region in (1): a region dominated by resonances over a non-resonant background, and a region where perturbative QCD is reliable. This separation is due to chiral symmetry breaking and confinement in the QCD vacuum, on the one side, and to the asymptotic freedom of QCD, on the other one.

As we move towards larger values of p^2, the non-resonant background grows due to the opening of new decay channels and resonances becoming broader due to phase space considerations; eventually all resonances disappear melting in a continuum. However, this continuum does not necessarily agree with the one predicted by perturbation theory (in particular, lowest order perturbation theory) if the value of p^2 where the melting of resonances occurs is too low.

Thus in addition, there is a third, intermediate, region (usually termed as a "duality violation region") where neither resonance dominance or perturbative physics describe well the data; resonances are nearly invisible in the continuum of multiparticle contributions and perturbation theory is still unreliable. However, it is crucial to note that large N_c counting rules indicate that the multiparticle background at a given (fixed) momentum must disappear in the large N_c limit.

At this point we need to be more definite about how we count resonances. In order to do this we introduce an 'error bar' in the magnitude of the correlators $|\Delta\Pi/\Pi| \sim \epsilon$ uniform over p^2. Resonances of (relative) height lower than ϵ over the background will be counted as part of the continuum, whereas those that stand out higher than ϵ will be retained as visible. The quantity ϵ will be the same for both VV and AA correlators. For a given ϵ and N_c one can find a finite number of visible resonances and establish an upper bound $p^2 \leqslant \Lambda_R^2$ above which one deals with continuum generated by intermediate multiparticle states but not resolved into visible resonances.

At high energies one expects quark–hadron duality to hold [24] and perturbation theory to provide accurate predictions with a (relative) precision ϵ down to a scale Λ^2_{PT}. By construction it is clear that $\Lambda_R < \Lambda_{PT}$. At intermediate values $\Lambda_R^2 < p^2 < \Lambda^2_{PT}$ the non-resonant multihadron picture previously described is adequate.

Now let us increase the number of colors while keeping ϵ fixed. According to the usual large N_c counting rules we expect that:

(a) The overall scale of the correlator (1) increases as N_c.
(b) Resonances become narrower and more distinct at a given momentum range, showing clearer Breit–Wigner shapes with widths $\sim 1/N_c$ and increasing their peaks $\sim N_c$. Their position, on the contrary, are independent of N_c at leading order.
(c) At a given value of p^2 the number of possible intermediate multiparticle (massive) states is fixed by kinematics, but their coupling constants behave as inverse powers of N_c and consequently the non-resonant hadron background decreases.

Let us now imagine drawing a band of (relative) width ϵ around the perturbative and non-resonant contributions in their respective regions of validity. As a consequence of (a)–(c), for fixed p^2 and a fixed value of ϵ more and more resonances stand out of the band as we increase the number of colors. Because we assume a Regge-like behavior for the radially excited states in the different channels appropriate for large meson masses, $(m_n)^2 \simeq (m_0)^2 + an$, $n \gg 1$, with $a \simeq 1.2\ \text{GeV}^2$, linearly rising trajectories imply that Λ_R^2 grows linearly with the number of visible resonances. Consequently, $\Lambda_R(N_c) \leqslant \Lambda_R(N_c')$ if $N_c < N_c'$. Correspondingly, at the values of p^2 where resonances disappear into the continuum, perturbation theory becomes more reliable as N_c increases.

Furthermore, the non-resonant, non-perturbative background decreases as N_c increases. In particular, the non-resonant background is getting suppressed with respect to the perturbative contribution at a given value of p^2. By increasing N_c it can be made arbitrarily small.

We shall show in Section 4 that large N_c counting rules imply that the number of visible resonances increases for a fixed value of ϵ linearly with N_c, so the region of validity of perturbation theory is reached rather quickly as N_c grows. Combined with the disappearance of the non-perturbative background at large N_c and continuity arguments, it becomes evident that for any value of ϵ there should be a value of N_c large enough (but still finite) where $\Lambda_R \simeq \Lambda_{PT}$. Indeed, with a given precision ϵ, perturbative estimations of continuum must be good for a finite number of open channels whereas the range of resonance physics must increase until these two scales meet each other. Thus there is no room for the "duality violation region" for N_c large enough.

If we accept this consequence of large N_c arguments, we can replace $\Pi_{\mu\nu}(p^2)$ by $\Pi^{PT}_{\mu\nu}(p^2)$ (the correlator calculated in perturbation theory) with an error bounded by ϵ for values of p^2 beyond the last visible resonance.

It is clear that none of vector resonances saturating the VV correlator in (1) should a priori coincide in mass with the corresponding AA one. We have to introduce a label $j = V, A$ in quantities such as m_n^j or a^j, but also in Λ_R^j or Λ_{PT}^j. However, the chiral symmetry of QCD for massless quarks guarantees the coincidence of VV and AA correlators at sufficiently high momenta in perturbation theory. Up to which point is perturbation theory valid though? For large (Minkowskian) values of p^2 perturbation theory has to be complemented with non-perturbative correction (counterpart of the OPE valid in the deep Euclidean region), which are different for the V and A channels, but are suppressed by powers of the momentum scale. In fact these non-perturbative corrections, that are suppressed by inverse powers of momenta, are also suppressed by powers of N_c as well as $p^2 \geqslant \Lambda_{PT}^2 \simeq \Lambda_R^2 \sim N_c$. Thus, at leading order in N_c, Λ_{PT}^j should actually be identical for the VV and AA channels and we can omit here the index j.

In order to substantiate more this claim notice that if the OPE, valid in the deep Euclidean region, induces via dispersion relations corrections to $\Pi^{PT}_{\mu\nu}(p^2)$ proportional to the four-quark condensate in the physical Minkowski region then, on dimensional grounds, these corrections are down by a power of $1/(p^2)^3$ and therefore in the kinematic region we are considering are of order $1/N_c$, to be compared with the leading perturbative contribution of order N_c^2 (since $(\Lambda_{PT})^2 \sim N_c$). Thus the difference between the VV and AA channels is enormously suppressed in the large N_c limit in the region of transition between resonance domination and perturbation theory and the argument in the previous paragraph holds.

Therefore, with a precision $1/N_c$, $\Lambda_{PT}^V = \Lambda_{PT}^A$. On the other hand, there is no a priori reason why Λ_R^V should be equal to Λ_R^A owing to chiral symmetry breaking. However, one has to take into account the characteristic scale of chiral symmetry breaking. This is related to the ratio

$$\Lambda_{\mathrm{CSB}} \sim \frac{\langle \bar{q}q\rangle}{F_\pi^2} \sim \frac{m_\pi^2}{2m_q} \sim 1\ \mathrm{GeV},$$

and does not depend on N_c at the leading order. Thus we can conclude from this low energy argument that the difference is, at most, of $\mathcal{O}(1)$ in the large N_c expansion. In addition, the fact that as we have seen, with a precision $1/N_c$, $\Lambda_R \simeq \Lambda_{PT}$ leads us to conclude that the difference $\Lambda_R^V - \Lambda_R^A$ is a decreasing function of N_c. Assuming that $(\Lambda_R^V)^2 - (\Lambda_R^A)^2 \sim \Lambda_{\mathrm{CSB}}^2 = \mathcal{O}(1)$ is however sufficient for our purposes.

The number of visible resonances need not be a priori the same in both channels either. Let N^V and N^A be the numbers of such resonances (visible with precision ϵ) for a given N_c. If linear Regge trajectories are appropriate for large meson masses, $(m_n^j)^2 \simeq (m_0^j)^2 + a^j n$, $n \gg 1$, then evidently $N^j \sim (\Lambda_R^j)^2/a^j$ and increases with growing N_c as the slopes a^j do not depend on N_c. One of our goals is to determine whether the slopes a^V and a^A differ in the large limit N_c or QCD dictates them to coincide.

3. Enter Weinberg sum rules

We shall now make use of Weinberg sum rules [22,23]. Let us take the chiral limit and impose the local current conservation,

$$\Pi^j_{\mu\nu}(p^2) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\Pi^j(p^2). \tag{2}$$

Then one can formally derive the spectral representation

$$\Pi^j(p^2) = -p^2 \int_0^\infty \frac{ds}{s} \frac{\rho^j(s)}{(p^2 - s + i\varepsilon)} + \Pi^j(0),$$
$$\rho^j(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi^j(s) > 0, \tag{3}$$

where $\rho^j(s)$ is related to the probability of producing particles with invariant mass squared s. One has to define the constants $\Pi^j(0)$ in order to remove a possible massless pole in the vector channel and to reproduce the pion pole in the axial one

$$\Pi^V(0) = 0, \qquad \Pi^A(0) = -F_\pi^2. \tag{4}$$

It is well known that the spectral representation (3) is formal, being UV divergent as the probability $\rho^j(s)$ does not decrease at very large s, being eventually saturated by the imaginary part of the perturbative decay amplitude into quarks which increases linearly with s. As to the IR pole $1/s$ the absence of other massless particles but the pion and the Adler zeroes in the chiral limit guarantee the IR integrability of $\rho^j(s)$. Thus the dispersion relations (3) need subtractions of the short-distance singularities. On the other hand, the Wilson analysis of OPE for correlators in x space allows to locate the singularities on the light cone which are perturbative due to asymptotic freedom and equivalent for vector and axial-vector channels [22,23]. Owing to this fact one can combine unambiguously the difference of VV and AA correlators to eliminate those singularities and derive two well convergent Weinberg sum rules

$$\int_0^\infty ds \frac{\rho^V(s) - \rho^A(s)}{s} = F_\pi^2, \tag{5}$$

$$\int_0^\infty ds \big(\rho^V(s) - \rho^A(s)\big) = 0. \tag{6}$$

We now consider these sum rules for a finite but large value of N_c and assume that $\Lambda_R^V \geqslant \Lambda_R^A$ (the reverse case can be treated similarly and leads to the same results). Let us saturate the entire spectral density $\rho^V(s) - \rho^A(s)$ by well-separated resonances up to $s = (\Lambda^A)^2$, by resonances for $\rho^V(s)$ and by perturbation theory for $\rho^A(s)$ when $(\Lambda^A)^2 < s < (\Lambda^V)^2$, as well as by perturbation theory for $\rho^{V,A}(s)$ (see [25] and references therein) when $(\Lambda^V)^2 \leqslant s$ with

$$\rho_{PT}(s) = N_c s C(s),$$
$$C(s) \equiv C_0\left(1 + \frac{3N_c\alpha_s(s)}{8\pi} + \cdots\right), \quad C_0 \equiv \frac{1}{24\pi^2}. \tag{7}$$

As previously indicated, non-perturbative contributions can be safely neglected because $(\Lambda_R^{(V,A)})^2$ are proportional to N_c and non-perturbative OPE-like contributions are suppressed by inverse powers of the scales. This is equivalent to stating that the number of colors is so large that with an ϵ precision the overlapping resonances are so wide that their multiparticle decays can be replaced by continuum and inclusively calculated by means of perturbation theory.

Then, to leading order of perturbation theory and the Weinberg sum rules (5) and (6) read

$$\sum_{n=0}^{N^V} \big(F_n^V\big)^2 - \sum_{n=0}^{N^A} \big(F_n^A\big)^2 = F_\pi^2 + N_c C_0\big(\big(\Lambda^V\big)^2 - \big(\Lambda^A\big)^2\big), \tag{8}$$

$$\sum_{n=0}^{N^V} \big(F_n^V\big)^2 \big(m_n^V\big)^2 - \sum_{n=0}^{N^A} \big(F_n^A\big)^2 \big(m_n^A\big)^2 = \frac{1}{2} N_c C_0\big(\big(\Lambda^V\big)^4 - \big(\Lambda^A\big)^4\big), \tag{9}$$

where we have used the fact that for separated narrow Breit–Wigner resonances one can calculate their individual contributions

$$\pi\rho_n^j(s) = \frac{(F_n^j m_n^j)^2\ \Gamma_n^j m_n^j}{(s - (m_n^j)^2)^2 + (\Gamma_n^j m_n^j)^2}, \tag{10}$$

extrapolating the integration to infinity and the result is independent of the width.

The perturbative contribution of course cancels between the V and A channels for $(\Lambda^V)^2 \leqslant s$ in the perturbative region. In Eqs. (8) and (9) we have approximated $C(s) \simeq C_0 = C(\infty)$ relying on asymptotic freedom. Possible corrections from this can be estimated as $\mathcal{O}(1/\log N_c)$ and are not difficult to compute.

4. On the saturation of spectral sum rules by resonances

Let us first prove that $N^j \sim N_c$ (we have used this result in the previous section). At larger N_c one observes the narrowing and growing of resonances, but they become progressively less marked at higher values of n. Resonances become invisible when the resonance width $m_n^j \Gamma_n^j$ becomes comparable with the distance between neighbor resonances (see similar arguments in [26]). For linear trajectories, in the Regge description of mesons [1]

$$\Gamma_n^j \sim \frac{B^j m_n^j}{N_c}.$$

Thus resonances in a given channel overlap when their widths $m_n^j \Gamma_n^j$ are comparable to the corresponding slopes

$$m_n^j \Gamma_n^j \sim \frac{B^j (m_n^j)^2}{N_c} \sim a^j,$$

i.e. for

$$N^j \sim N_c/B^j,$$

showing that the number of visible resonances in each channel is proportional to N_c as previously announced. This corresponds to $(\Lambda_R^j)^2 \sim N^j a^j \sim N_c a^j / B^j$.

The corresponding maxima at the location of the resonances are given by

$$\pi \rho_n^j(s)\big|_{s=(m_n^j)^2} = \frac{(F_n^j)^2 m_n^j}{\Gamma_n^j}. \tag{11}$$

A more precise quantitative estimation of $(\Lambda_R^j)^2$ or N^j for a fixed ϵ is difficult as the additive Breit–Wigner description of individual resonances is not reliable when there is substantial overlap. Nevertheless a semi-quantitative estimate, which undoubtedly captures the right N_c behavior, can be done: let us determine N^j by demanding that oscillations due to resonances relative to the background be $\sim \epsilon$. The value of the minimum between two adjacent resonances (in the Breit–Wigner approximation) is reached for

$$\bar{s}_n \simeq \frac{1}{2}\left(\left(m_{n-1}^j\right)^2 + \left(m_n^j\right)^2\right).$$

Plugging this value into $\rho_n^j(s)$ we get

$$\pi \rho_n^j(s)\big|_{s=\bar{s}_n} \simeq \frac{(F_n^j)^2 (N^j)^2 \frac{B^j}{N_c}}{\frac{1}{4} + \frac{(N^j B^j)^2}{N_c^2}}. \tag{12}$$

Comparing this with (11), we get $N^j \simeq N_c / 2\sqrt{\epsilon} B_j$, which is a rather interesting expression.

The Regge model also implies asymptotically equal decay constants; that is, $F_n^j \sim F^j$, $n \gg 1$. At the point where resonances become invisible for N_c large enough, the spectral density levels off at a value that must match (with precision ϵ) that of perturbation theory

$$\pi \rho_n^j(s)\big|_{s=(\Lambda_R^j)^2} = \frac{(F_n^j)^2 (\Lambda_R^j)^2}{a^j} \simeq N_c C_0 (\Lambda_{PT})^2. \tag{13}$$

Therefore $(F^j)^2 \simeq N_c C_0 a^j$. Assuming that $(\Lambda^V)^2 - (\Lambda^A)^2$ is at most of $\mathcal{O}(1)$ in the large N_c expansion, the Weinberg sum rules lead immediately to the conclusion that $F^V \simeq F^A$ at leading order in N_c because otherwise in the first Weinberg sum rule (8)

$$\sum_{n=0}^{N^V} \left(F_n^V\right)^2 - \sum_{n=0}^{N^A} \left(F_n^A\right)^2 \sim N_c\left(\left(F^V\right)^2 - \left(F^A\right)^2\right) \sim N_c^2,$$

whereas it should be of $\mathcal{O}(N_c)$ as is clear by looking at the r.h.s. of Eq. (8).

These results can be easily improved with the inclusion of few orders of perturbation theory at the expense of the decay coupling constants F_n^j dependence on the radial number. Namely, for $n \gg 1$ one gets in the NLO asymptotics

$$\left(F_n^j\right)^2 \simeq N_c C(a_j n) a_j, \qquad C(s) = C_0\left(1 + 3N_c \alpha_s(s)/8\pi + \cdots\right).$$

However, for the sake of clarity, in what follows we retain the leading perturbative order only.

Next let us analyze the linear Regge asymptotics for radial excitations. Consider meson states lying on the trajectories asymptotically, $(m_n^j)^2 \simeq (m_0^j)^2 + a^j n$, $n \gg 1$. Then, if $F^{V,A} \simeq F$, from the second Weinberg sum rule (9) we get $a^V \simeq a^A \equiv a$, i.e. the slope of trajectories is universal. Indeed, let us write $(\Lambda^j)^2 \simeq N^j a^j + (\Lambda_0^j)^2$. Then up to terms subleading in N_c, equating both sides of the second sum rule,

$$\begin{aligned}
&\left(F^V\right)^2\left(\frac{1}{2} N^V\left(N^V+1\right) a^V + N^V\left(m_0^V\right)^2\right) \\
&\quad - \left(F^A\right)^2\left(\frac{1}{2} N^A\left(N^A+1\right) a^A + N^A\left(m_0^A\right)^2\right) \\
&\quad \simeq \frac{1}{2}\left(N^V\right)^2 (F)^2 \left(a^V - a^A\right) \\
&\quad = \frac{1}{2} N_c C_0\left[\left(N^V a^V + \left(\Lambda_0^V\right)^2\right)^2 - \left(N^A a^A + \left(\Lambda_0^A\right)^2\right)^2\right] \\
&\quad \simeq \frac{1}{2} N_c C_0 \left(N^V\right)^2\left(\left(a^V\right)^2 - \left(a^A\right)^2\right),
\end{aligned} \tag{14}$$

which match each other iff $a^V = a^A$ for $(F^j)^2 \simeq N_c C_0 a^j$.

Finally, from (14) and from the relation $F^2 = N_c C_0 a$ it follows that

$$\left(m_0^V\right)^2 - \left(m_0^A\right)^2 \simeq \left(\Lambda_0^V\right)^2 - \left(\Lambda_0^A\right)^2 \tag{15}$$

at leading order. Indeed, the second sum rule is saturated by

$$N^V F^2\left(\left(m_0^V\right)^2 - \left(m_0^A\right)^2\right) \simeq N^V N_c C_0 a\left(\left(\Lambda_0^V\right)^2 - \left(\Lambda_0^A\right)^2\right),$$

whereas other terms are finite. Thus a possible finite shift between mass spectra of mesons with different parities in the large N_c approximation has to be accompanied with the same shift in cutoffs for resonance regions, even though as we have argued we expect this difference to be subleading in N_c. However a deviation from the universality $a^V = a^A$ may then also give a comparable term.

$$\left(N^V\right)^2 a^V - \left(N^A\right)^2 a^A \sim \left(N^V\right)^2 \Delta a / N_c \sim N^V \Delta a.$$

We stress once more that the previous results are not based on the number of resonances N^V, N^A or cutoff Λ_R^V, Λ_R^A being equal. We do expect however that their difference is subleading in N_c for the reasons given above.

Similar arguments can be applied to other pairs of channels with equal quantum numbers but parity.

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References

[1] M. Shifman, in: Quark–Hadron Duality and the Transition to pQCD, World Scientific, Singapore, 2006, pp. 171–192, hep-ph/0507246.
[2] A.A. Andrianov, D. Espriu, R. Tarrach, Nucl. Phys. B 533 (1998) 429;
A.A. Andrianov, D. Espriu, JHEP 9910 (1999) 022;
A.A. Andrianov, V.A. Andrianov, hep-ph/9911383;
V.A. Andrianov, S.S. Afonin, Eur. Phys. J. A 17 (2003) 111.
[3] S.R. Beane, Phys. Rev. D 64 (2001) 116010.
[4] S.S. Afonin, Phys. Lett. B 639 (2006) 258;
S.S. Afonin, Nucl. Phys. B 779 (2007) 13.
[5] S.S. Afonin, A.A. Andrianov, V.A. Andrianov, D. Espriu, JHEP 0404 (2004) 039;
S.S. Afonin, A.A. Andrianov, V.A. Andrianov, D. Espriu, Int. J. Mod. Phys. A 21 (2006) 885;
S.S. Afonin, A.A. Andrianov, V.A. Andrianov, D. Espriu, Nucl. Phys. B (Proc. Suppl.) 164 (2007) 296.
[6] L.Y. Glozman, Phys. Lett. B 587 (2004) 69;
L.Y. Glozman, Int. J. Mod. Phys. A 21 (2006) 475.
[7] L.Y. Glozman, Phys. Rep. 444 (2007) 1.
[8] E.S. Swanson, Phys. Lett. B 582 (2004) 167.
[9] S.S. Afonin, D. Espriu, JHEP 0609 (2006) 047.
[10] E. Ruiz Arriola, W. Broniowski, Phys. Rev. D 73 (2006) 097502.
[11] M. Golterman, S. Peris, JHEP 0101 (2001) 028;
M. Golterman, S. Peris, Phys. Rev. D 67 (2003) 096001;
M. Golterman, S. Peris, Phys. Rev. D 74 (2006) 096002.
[12] O. Catà, M. Golterman, S. Peris, Phys. Rev. D 74 (2006) 016001;
O. Catà, M. Golterman, S. Peris, JHEP 0508 (2005) 076.
[13] J.J. Sanz-Cillero, Nucl. Phys. B 732 (2006) 136.
[14] M. Shifman, A. Vainshtein, arXiv: 0710.0863 [hep-ph].
[15] R. Casero, E. Kiritsis, A. Paredes, Nucl. Phys. B 787 (2007) 98.
[16] J. Mondejar, A. Pineda, JHEP 0710 (2007) 061.
[17] S.S. Afonin, arXiv: 0710.4921 [hep-ph].
[18] A.B. Kaidalov, Surv. High, Energy Phys. 13 (1999) 265.
[19] D.V. Bugg, Phys. Rep. 397 (2004) 257.
[20] S.S. Afonin, Eur. Phys. J. A 29 (2006) 327;
S.S. Afonin, Int. J. Mod. Phys. A 22 (2007) 4537;
S.S. Afonin, Mod. Phys. Lett. A 22 (2007) 1359;
S.S. Afonin, Phys. Rev. C 76 (2007) 015202.
[21] E. Klempt, A. Zaitsev, Phys. Rep. 454 (2007) 1.
[22] S. Weinberg, Phys. Rev. Lett. 18 (1967) 507;
S. Weinberg, Phys. Rev. 16 (1968) 1568.
[23] C. Bernard, A. Duncan, J. LoSecco, S. Weinberg, Phys. Rev. D 12 (1975) 792.
[24] M.A. Shifman, A.I. Vainstein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385;
M.A. Shifman, A.I. Vainstein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 448.
[25] S.G. Gorishny, A.L. Kataev, S.A. Larin, Phys. Lett. B 259 (1991) 144.
[26] A. Casher, H. Neuberger, S. Nussinov, Phys. Rev. D 20 (1979) 179;
B. Blok, M.A. Shifman, D.X. Zhang, Phys. Rev. D 57 (1998) 2691;
B. Blok, M.A. Shifman, D.X. Zhang, Phys. Rev. D 59 (1999) 019901, Erratum;
M. Shifman, hep-ph/0009131;
M. Shifman, Czech. J. Phys. 52 (2002) B102.